

Critical behavior of a lattice prey-predator model

Tibor Antal and Michel Droz

Département de Physique Théorique, Université de Genève, CH 1211 Genève 4, Switzerland

Adam Lipowski

Department of Physics, A. Mickiewicz University, 61-614 Poznań, Poland

Géza Ódor

Research Institute for Technical Physics and Materials Science, P.O. Box 49, H-1525 Budapest, Hungary

(Received 17 April 2001; published 29 August 2001)

The critical properties of a simple prey-predator model are revisited. For some values of the control parameters, the model exhibits a line of directed percolationlike transitions to a single absorbing state. For other values of the control parameters one finds a second line of continuous transitions toward an infinite number of absorbing states, and the corresponding steady-state exponents are mean-field-like. The critical behavior of the special point T (bicritical point), where the two transition lines meet, belongs to a different universality class. A particular strategy for preparing the initial states used for the dynamical Monte Carlo method is devised to correctly describe the physics of the system near the second transition line. Relationships with a forest fire model with immunization are also discussed.

DOI: 10.1103/PhysRevE.64.036118

PACS number(s): 05.70.Ln, 64.60.Cn, 82.20.-w

I. INTRODUCTION

The study of prey-predator systems has attracted much attention since the pioneering works of Lotka [1] and Volterra [2]. Working at a mean-field level (homogeneous populations) they showed that, depending on the initial state, the system can evolve toward a simple steady state or a limit cycle, in which the populations oscillate periodically in time.

An important question is the understanding of the role played by the local environment on the dynamics (spatial effects) [3], and, accordingly, many extended prey-predator models have been studied during the past years [4–9]. Recently, a simple prey-predator model was introduced by some of us [10]. Although governed by only two control parameters, this model exhibits a rich phase diagram. As a function of the two control parameters λ_a and λ_b , which are the growth rates of the prey and predator, respectively, two different phases are observed: a pure prey phase (P), and a coexistence phase of prey and predator in which an oscillatory (O) region and a non-oscillatory (NO) region can be distinguished. For a system size $L \rightarrow \infty$, these three different domains meet at a particular point, called $T = (\lambda_a^T, \lambda_b^T)$ (precise definitions are given below). It was shown [10] that $\lambda_a^T = 0$ and $\lambda_b^T \approx 5.0 \pm 0.3$. For $\lambda_a > 0$, a phase transition line between the pure prey phase and the coexistence phase is present, and the critical exponents along this line are the ones of directed percolation (DP) [11]. However, it was also observed that when the growth rates of the prey are $\lambda_a \rightarrow 0$ and $\lambda_b > \lambda_b^T$, the model undergoes a non-DP continuous phase transition. Since DP is a generic universality class for models with absorbing states (unless some special conditions are satisfied [12]), the existence of such a transition is certainly surprising. These two lines of different continuous nonequilibrium phase transitions meet at the bicritical point T [13], and one can forecast that the critical behavior at this particu-

lar point may also belong to an interesting universality class.

The goal of this paper is to study in more detail the properties of these non-DP phase transitions. We performed extensive steady-state simulations, which confirm the non-DP character of the transition in the limits $\lambda_a \rightarrow 0$ and $\lambda_b \geq \lambda_b^T$.

As already shown [10], in this limit the model exhibits an oscillatory behavior. However, in addition to this, for $\lambda_a = 0$ the model has infinitely many absorbing states. These two properties are responsible for a rather peculiar behavior of the model, which becomes particularly transparent when the model is examined using the dynamical Monte Carlo method. When applied to models with infinitely many absorbing states, this dynamical method uses so-called natural absorbing states, which are the most likely states to be reached by the dynamical evolution of the system. We show, however, that this common but somehow heuristic procedure fails here. Indeed, for the present model, natural absorbing states contain only short-ranged islands of prey on which spreading is not critical. To restore the criticality of the spreading we generated the absorbing states using a quasi-static approach to the critical point. This example shows that, for some models with infinitely many absorbing states, a special approach is needed to examine the dynamical properties of the critical point.

The paper is organized as follows. In Sec. II, the model is defined, and some of its properties are discussed. A thorough investigation of the critical behavior, using two different complementary approaches, is reported. In Sec. III, the critical behavior is investigated using steady-state properties while in Sec. IV one uses a dynamical Monte Carlo method. It is shown that for $\lambda_a \rightarrow 0$ and $\lambda_b \geq \lambda_b^T$ the steady-state exponents are indeed mean-field-like, while the dynamical exponents are nonuniversal, depending continuously upon λ_b . Nevertheless, a trace of the mean-field character of the transition shows up in the scaling relations among dynamical exponents. The critical behavior at point T is also investi-

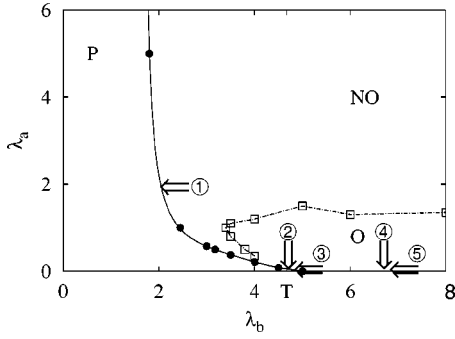


FIG. 1. Phase diagram obtained by extrapolation of the simulation results to the case $L \rightarrow \infty$. The solid line represents the DP transition between the prey phase (P) and the nonoscillatory part (NO) of the coexistence phase (The \bullet symbols are the simulated values, while the solid line is just a guide to the eyes). The \square symbols delimit the crossover between the oscillatory (O) and nonoscillatory (NO) regimes present in the coexistence phase. The arrows correspond to the path described in the text, along which the critical exponents have been measured. Paths 2 and 3 end at the point T on the horizontal axis.

gated, and it turns out that the corresponding exponents belong to an interesting universality class. Finally, physical arguments explaining the above findings are given in Sec. V.

II. MODEL

The model used in Ref. [10] is defined as follows. Each cell of a two-dimensional square lattice (of size $L \times L$, with a periodic boundary condition), labeled by the index i , can be, at time t , in one of the three following states: $\sigma_i = 0, 1$, and 2 . A cell in state $0, 1$, or 2 corresponds, respectively, to a cell which is empty, occupied by prey, or simultaneously occupied by prey and predators. The transition rates for site i are (i) $0 \rightarrow 1$ at rate $\lambda_a(n_{i,1} + n_{i,2})/4$, (ii) $1 \rightarrow 2$ at rate $\lambda_b(n_{i,2})/4$, and (iii) $2 \rightarrow 0$ at rate 1 , where $n_{i,\sigma}$ denotes the number of nearest neighbor sites of i which are in the state σ . The first two processes model the spreading of prey and predators. The third process represents the local depopulation of a cell due to overly greedy predators. The rate of the third process is chosen to be 1 , which sets the time scale; hence t , as well as λ_a and λ_b , are dimensionless quantities.

The properties of this model have been investigated both by mean-field and Monte Carlo methods [10]. The Monte Carlo result extrapolated to the case $L \rightarrow \infty$ is summarized in Fig. 1. The transition line between the prey phase and the coexistence phase, $\lambda_b^*(\lambda_a)$, belongs to the directed percolation universality class [11], as expected, and terminates at the point $T \equiv (\lambda_a^T = 0, \lambda_b^T)$, where the P , O , and NO domains meet. For $\lambda_b > \lambda_b^T$, the transition between the oscillatory domain of the coexistence phase and the prey phase takes place at $\lambda_a = 0$. Along this transition line the predator density approaches zero as a power law $b \propto (\lambda_a)^{\beta_2}$, with $\beta_2 \approx 1$, and so does not belong to the DP class, which is somehow unexpected. The value $\beta_2 \approx 1$ lead to the conjecture [10] that this second transition could be mean-field-like. There is a crossover between the O and NO parts of the coexistence phase.

The purpose of the present study is to give a complete description of the nature of the transitions near the line $\lambda_a = 0$ for $\lambda_b \geq \lambda_b^T$.

It is worth to mention, that our model is closely related to a model introduced by Drossel and Schwabl [14] to investigate the effect of immunization in an extension of the simple forest-fire model [15]. That three-state model (0 is an empty site, 1 is a tree, and 2 is a burning tree) differs from our model in some details: the growth rate of a tree ($\sigma: 0 \rightarrow 1$) is p , independently of the environment, and a tree is ignited ($\sigma: 1 \rightarrow 2$) with rate $(1-g)\Theta(n_2)$, (Θ is the usual Heaviside function). This second process models the immunization of trees against fire. The third process ($\sigma: 2 \rightarrow 0$) occurs at rate 1 . For nonzero immunity and $p > 0$, Albano [16] showed that a transition toward a single absorbing state is DP like, while for $p = 0$ (at the end point of the DP transition line), the transition belongs to the dynamical percolation universality class, and the absorbing state is not unique.

III. STEADY STATE STUDY OF THE CRITICAL BEHAVIOR

Extensive Monte Carlo simulations for system sizes up to 4000×4000 have been performed to investigate the behavior of the predator density b , for $\lambda_a \rightarrow 0$ and three different values of λ_b , namely, $\lambda_b = 4.67, 5.0$, and 6.0 , following trajectories of types 2 and 4 in Fig. 1. The value $\lambda_a = 0, \lambda_b = 4.67$ corresponds to the best determination of the end point T , obtained by the dynamical approach described below. Owing to the oscillatory behavior near the critical line ($\lambda_a = 0, \lambda_b > \lambda_b^T$), the system very easily evolves into an absorbing state, where the predators are extinct; therefore, a careful initialization is needed in the simulations. Usually we did 10^4 Monte Carlo steps (MCS's) to approach the desired value of λ_a , continuously decreasing it from an initial value. Then 10^4 MCS's were used to reach the stationary state, in which the densities and the fluctuations of prey and predators were averaged over $\approx 2 \times 10^5$ MCS's for each λ_a or λ_b point. It is found that $b \propto (\lambda_a)^{\beta_2}$ for $\lambda_a \rightarrow 0$. In order to see corrections to scaling, we compute the effective exponent

$$\beta_{eff}(\lambda_a(i)) = \frac{\ln b[\lambda_a(i)] - \ln b[\lambda_a(i-1)]}{\ln \lambda_a(i) - \ln \lambda_a(i-1)}. \quad (1)$$

where $\lambda_a(i)$ and $\lambda_a(i-1)$ are two consecutive values of the control parameter λ_a .

As shown in Fig. 2, for $\lambda_b = 5$ and 6 , the linear extrapolation of β_{eff} converges to ≈ 1 within statistical errors:

$$\beta_2(\lambda_b = 5.0) = 1.01(1), \quad (2)$$

$$\beta_2(\lambda_b = 6.0) = 0.96(4). \quad (3)$$

For $\lambda_b = 4.67$ (a trajectory of type 2 in Fig. 1) it goes to a somewhat higher value, and one finds

$$\beta_2(\lambda_b = 4.67) = 1.33(4). \quad (4)$$

The measurement of the fluctuations,

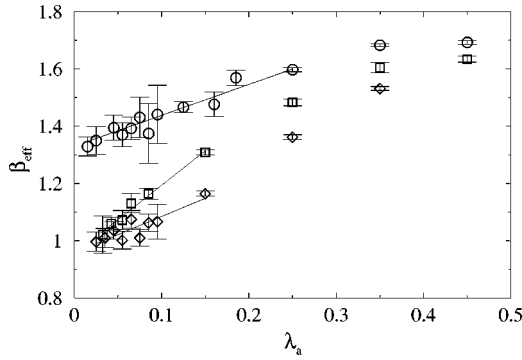


FIG. 2. Predator density critical exponent β_2 obtained for several values of λ_a and $\lambda_b=4.67$ (\circ), 5.0 (\square), and 6.0 (\diamond).

$$\chi_b = L^2 \langle (b - \langle b \rangle)^2 \rangle \propto (\lambda_a)^{-\gamma}, \quad (5)$$

are less precise, and we estimate $\gamma = -0.6(16)$ at $\lambda_b = 4.67$, and $\gamma \approx 0$ for $\lambda_b = 5$ and 6 .

For $\lambda_b > \lambda_b^T$ these values are consistent with the previous prediction [10] $\beta_2 \approx 1$. However, at the bicritical point T the value of β_2 is completely different and thus belongs to a new universality class.

IV. DYNAMICAL STUDY OF THE CRITICAL BEHAVIOR

A very useful technique to study the critical properties of a system with absorbing states is the so-called dynamical Monte Carlo method [17]. In this approach, the system is prepared in an initial state, which is one of the absorbing states up to one site, which is set to be in the active state. One considers an ensemble of trials starting from the same initial state. Certain dynamical quantities exhibit a power law behavior when the system is critical. For example, the survival probability behaves as

$$P(t) \propto t^{-\delta}. \quad (6)$$

The deviation from this power law behavior, when the system is off-critical, provides a very precise way to locate the critical point.

The number of active sites $N(t)$ behaves as

$$N(t) \propto t^\eta \quad (7)$$

while, for the mean square spreading from the origin $R^2(t)$,

$$R^2(t) \propto t^z, \quad (8)$$

where the dynamical exponent $z = 2\nu_\perp / \nu_\parallel$ is the ratio of the critical exponents of spatial (ν_\perp) and temporal (ν_\parallel) correlation lengths. Some scaling relations between these exponents can be also derived [18]. The results obtained by the dynamical Monte Carlo method are the following.

A. Case of $\lambda_a = 0$

First we simulated the model on the $\lambda_a = 0$ line, taking, as an absorbing state, a lattice filled with prey (trajectory of type 3 in Fig. 1). Measuring the survival probability $P(t)$, we found that $\lambda_b = \lambda_b^T \approx 4.67$ is the critical point, which sepa-

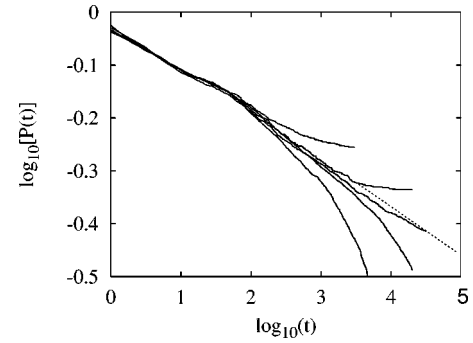


FIG. 3. The survival probability $P(t)$ as a function of t obtained for $\lambda_a = 0$ and (from the top) $\lambda_b = 4.6, 4.65, 4.67, 4.7,$ and 4.75 (trajectory of type 3 in Fig. 1). We used a system size $L = 3000$, and up to 10^5 independent runs were made for each value of λ_b . The dotted line corresponds to $\delta = 0.092$.

rates the absorbing phase ($\lambda_b < \lambda_b^T$) and the phase with annular growth ($\lambda_b > \lambda_b^T$). The measurement of the slope at $\lambda_b = \lambda_b^T$ (see Fig. 3) gives $\delta \approx 0.095(5)$, which is very close to the value obtained for dynamical percolation, for which, in two dimensions, $\delta = 0.092$ [18]. Moreover, using Eqs. (7) and (8) we obtained $\eta = 0.60(5)$ and $z = 1.72(4)$, which are also very close to the dynamical percolation values.

Note that the usual order parameter critical exponent β , defined for dynamical percolation as $b \propto (\lambda_b^T - \lambda_b)^\beta$, differs from our definition of β_2 in Sec. III. Thus it is not surprising that these two exponents differ. Also note that the dynamical estimation of the critical end point $\lambda_b = \lambda_b^T$ has been used in the static approach of Sec. III.

B. Case of $\lambda_a > 0$

The same scheme was used for $\lambda_a > 0$. The critical points were located for $\lambda_a = 0.5$ and 1 (a trajectory of type 1 in Fig. 1), and their values agree with the steady state results of Ref. [10]. Measuring the slope at criticality (see Fig. 4) we estimate $\delta \approx 0.45$, i.e., a value compatible with DP [18]. The

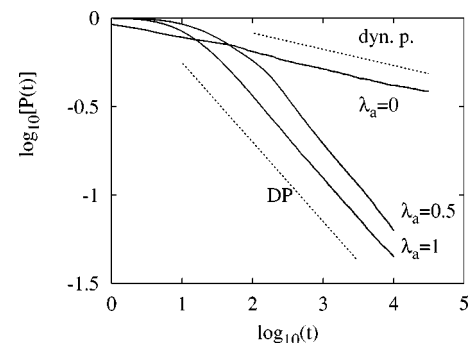


FIG. 4. The survival probability $P(t)$ as a function of t obtained for $\lambda_a = 0$ and $\lambda_b = 4.67$ (trajectory of type 3 in Fig. 1), $\lambda_a = 0.5$ and $\lambda_b = 3.175$, and $\lambda_a = 1$, $\lambda_b = 2.451$ (trajectory of type 1 in Fig. 1). Corresponding values of λ_a are also shown in the figure. For $\lambda_a > 0$ we used the system size $L = 1000$, and up to 10^5 independent runs were made for each curve. The dotted lines have a slope corresponding to the exponent δ of the dynamical and the directed percolation.

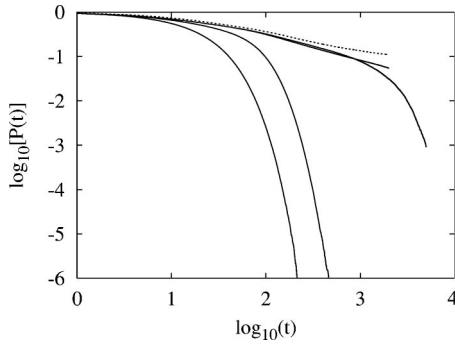


FIG. 5. The survival probability $P(t)$ as a function of t obtained for $\lambda_a=0$, and for $\lambda_b=6$ (solid line) and 8 (dotted line). Absorbing states were obtained using continuous cooling, with the cooling rates (from the top) $r=0.0001, 0.0001, 0.001, 0.01$, and ∞ . For the slowest cooling we used a system size $L=1000$. For each cooling rate we generated 10^3 absorbing states, and for each absorbing state we generated from 10^2-10^5 independent runs.

fact, that, for $\lambda_a>0$, the phase transition belongs to the DP universality class was already confirmed using the static calculations [10].

In Sec. II we already noted the similarity of the present model with a forest-fire model with immunization. Results presented in this section provide further arguments supporting such an analogy. Indeed, dynamical exponents measured by Albano for the forest fire model are also close to the dynamical percolation (without growth of tree) and the directed percolation (with growth of tree) [16].

C. Inhomogeneous absorbing states

Static simulations suggest that the model becomes critical on the line $\lambda_b>\lambda_b^T$ and $\lambda_a=0$. Moreover, let us note that for $\lambda_a=0$ there are infinitely many absorbing states: indeed, any configuration without predators is an absorbing state. It is well known that the dynamical Monte Carlo method can also be applied to models with infinitely many absorbing states. However, as we will see below, the applicability of this method to the criticality on this line requires serious reconsiderations.

First, let us recall that the dynamical Monte Carlo method for models with infinitely many absorbing states usually uses the so-called natural absorbing states, i.e., states which are reached by the model's dynamics. Numerical evidence suggests that for such states the dynamical critical point coincides with the static one. Moreover, the dynamical exponents δ and η , measured on such states, take universal values.

Following this prescription, we generated natural absorbing states for $\lambda_a=0$ and $\lambda_b>\lambda_b^T$, and then used such states to perform dynamical simulations. An initial configuration was chosen randomly, with equal probabilities for a site being empty, occupied by prey, or occupied simultaneously by prey and predator. Fixing $\lambda_a(=0)$ and λ_b , we then allowed the system to evolve until an absorbing configuration was reached (i.e., all predators die out). Our results, presented in Fig. 5, show, however, that the spreading of activity is not critical (i.e., power law), but rather exponential. But we can argue that this is not surprising. Indeed, a random initial

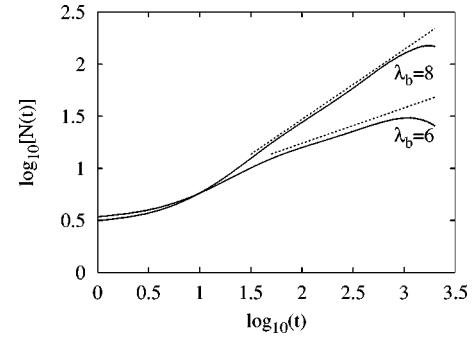


FIG. 6. The number of active sites $N(t)$ as a function of t obtained for $\lambda_a=0$ and $\lambda_b=6$ and 8. Absorbing states were obtained using continuous cooling, with a cooling rate $r=0.0001$. Straight dotted lines have slopes corresponding to $\eta=0.67$ and 0.35 .

configuration (with a probability of prey equal to $1/3$) is below the percolation threshold with respect to the clusters of prey, and contains only finite clusters of them [19]. On such clusters activity certainly last only for a finite time (for $\lambda_a=0$ prey do not grow), and the exponential decay of $P(t)$, seen in Fig. 5, is an expected feature.

Clearly, the lack of criticality in $P(t)$ is due to the finiteness of prey clusters in the natural absorbing states. In principle, we can cure this effect, starting from random initial configurations containing a larger fraction of prey. For sufficiently large concentrations the system will be above the percolation threshold, and the activity will spread forever. Probably, for a certain concentration of prey, we can tune the system to have a power-law decay for $P(t)$. Such a procedure, however, is somehow artificial, and the criticality of spreading will not be related with the static criticality of the system.

The question arises here of whether it is possible to generate absorbing states having the following properties: first, to be generated more “naturally,” and, second, to exhibit a critical spreading related to the steady state critical properties. In the following we suggest a procedure which imitates a quasistatic approach to the critical point on the line $\lambda_a=0$. In our approach we gradually reduce λ_a with time, according to the formula

$$\lambda_a(t) = \lambda_a^0 \exp(-rt), \quad (9)$$

where $\lambda_a^0=1$, and r is the “cooling” rate. [We expect that the detailed time dependence in Eq. (9) is not relevant as long as it is a slow process.] We stop cooling when an absorbing state is reached. When the cooling is slow, the system has enough time to build large clusters of prey. Our simulations for $\lambda_b=6$ suggest (see Fig. 5) that, in the limit $r\rightarrow 0$, such absorbing states are critical, with $\delta=0.59(10)$ (along trajectories of type 5 in Fig. 1). Measuring the number of active sites $N(t)$, and using Eq. (7), we estimate $\eta=0.34(10)$ (see Fig. 6). The departure of the curves from a straight line observed for large values of time is related to the finiteness of the cooling rate. Moreover, we measured the averaged squared distance $R^2(t)$, and, using Eq. (8), obtained $z=2.0(1)$ (see Fig. 7). Actually, we expect that the correct value of this exponent is $z=2$. Indeed, in Eq. (8) one aver-

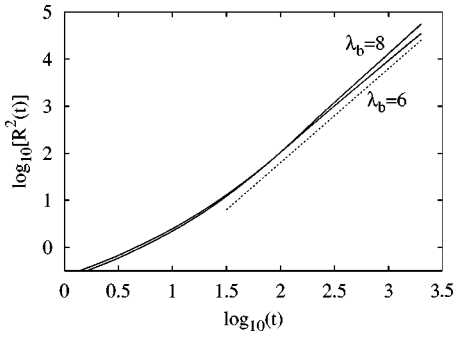


FIG. 7. The squared distance of active sites $R^2(t)$ as a function of t obtained for $\lambda_a=0$, $\lambda_b=6$, and 8. Absorbing states were obtained using continuous cooling with the cooling rate $r=0.0001$. The straight dotted line has a slope corresponding to $z=2$.

ages only over surviving runs; thus the long-time contributions to $R^2(t)$ come from rare events, when the activity happened to be placed on a large island of prey. Since $\lambda_b > \lambda_b^T$, the activity on such islands spreads in a deterministic way (annular growth), which leads to $z=2$.

The same procedure for $\lambda_b=8$ yields $\delta=0.35(10)$, $\eta=0.67(10)$, and $z=2.0(1)$ (relatively large errors of estimations of critical exponents are due to several, difficult to estimate, factors, such as finite time t , finite cooling rate r , and statistical fluctuations). Such results confirm that $z=2$, and suggest that exponents δ and η might change continuously with λ_b . The nonuniversality of these exponents is a well-known property of some other models belonging to the directed percolation universality class [20]. Note, however, that the situation is different in our case, because the nonuniversality is related to the value of λ_b rather than to the choice of the initial state. Such a control parameter dependence was already observed in other models [21]. Note that nonuniversal behavior is not present along the DP line, since the corresponding absorbing state is unique. Let us finally note that $\delta + \eta$ seems to be close to unity, which is an exact mean-field result ($\delta_{MF}=1, \eta_{MF}=0$). This is the only dynamical trace of the mean-field nature of the transition observed in the steady state. Let us emphasize, however, that the criticality of spreading appears only if we prepare the absorbing states using a method which mimics the quasi-steady-state evolution of the model.

TABLE I. Critical exponents around $\lambda_a=0$.

Exponent	$\lambda_b=4.67$	5.0	6.0	8.0
β_2	1.33(4)	1.01(1)	0.96(4)	
γ	-0.65(10)	-0.1(1)	-0.05(5)	
δ	0.095(5)		0.59(10)	0.35(10)
η	0.60(5)		0.34(10)	0.67(10)
z	1.72(4)		2.0(1)	2.0(1)

V. CONCLUSIONS

The detailed investigation of the critical properties of a simple prey-predator model introduced in Ref. [10] showed the presence of three different types of nonequilibrium phase transitions between active and absorbing states. First, the existence of a typical DP-like transition line was confirmed at $\lambda_b^*(\lambda_a)$ for $\lambda_a > 0$. Second, a mean-field-like transition was observed for $\lambda_a \rightarrow 0$, $\lambda_b > \lambda_b^T$. The mean-field character of this transition can be explained in terms of oscillations present in the model. Indeed, as described in Ref. [10], when $\lambda_a \rightarrow 0$, the system is subject to large density oscillations. These oscillations generate an important local mixing of the possible states, leading to a mean-field-like behavior. The criticality along the $\lambda_a=0$ line was confirmed with a dynamical approach using specially prepared inhomogeneous initial states. Some dynamical trace of the mean-field nature of this transition was also observed. Third, at the bicritical point T , where the two different critical lines meet, we found a dynamical percolation type transition moving along the $\lambda_a=0$ line, while, when approaching the point T from finite λ_a values, we observed an interesting type of critical behavior.

The measured exponents corresponding to the $\lambda_a=0$ line are summarized in Table I. The best numerical estimates for the critical exponents of the two-dimensional dynamical percolation are given for comparison: $\delta=0.092$, $\eta=0.586$, and $z=1.771$ [18].

This work was partially supported by the Swiss National Foundation, the Hungarian Academy of Sciences (Grant OTKA T-25286, T-029792 and Grant Bolyai BO/00142/99), and by the Polish State Committee for Scientific Research (5 P03B 032 20). The simulations were performed partially on the SZTAKI parallel computing cluster.

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