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REGULAR ARTICLE

Learning to save in a voluntary pension system: toward an agent-based model

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Abstract Mandatory pension systems partially replace old-age income, therefore the government matches additional life-cycle savings in a voluntary pension system. Though the individual saving decisions are apparently independent, the earmarked taxes (paid to finance the matching) connect them. Previous models either neglected the endogenous tax expenditures (e.g. Choi et al., in: Wise (ed) Perspectives in the economics of aging, University of Chicago Press, Chicago, pp 81–121, 2004) or assumed very sophisticated saving strategies (e.g. Fehr et al. in FinanzArchiv Pub Finance Anal 64:171–198, 2008). We create twin models: myopic workers learn (i) from farsighted workers using public information (analytic model) and (ii) also from each other (agent-based model). These models provide more realistic results on saving behavior and the impact of matching on the income redistribution than the earlier models.

Keywords Life-cycle savings · Overlapping generations · Mandatory pensions · Voluntary pensions · Agent-based models

JEL Classification D91 · H55

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1 Introduction

All over the developed world, governments operate *mandatory* pension systems to replace income and minimize old-age poverty. In general, the size of the mandatory system is low enough to leave room to be filled by a *voluntary* pension system. As a rule, participants of a voluntary system can only withdraw their voluntary savings after retirement, and as a compensation their savings enjoy tax advantages or matching. While there is a general agreement that this separation of mandatory and voluntary systems is socially advantageous, there are important debates about the qualitative as well as quantitative design.

To explain our contributions in a nutshell, we introduce the following concepts. In a complex economic system, the participants can use publicly available information and can also learn from directly observing their neighborhood. Concerning lifecycle saving, we may differentiate various degrees of shortsightedness: the less shortsighted a worker, the more she saves for her retirement. In a voluntary pension system the government matches the worker's voluntary new savings proportionally to a matching rate, at least up to a cap. To simplify the exposition we introduce a concept called *relative propensity to save* which is the ratio of the actual to the estimated optimal voluntary saving of shortsighted workers. Because of some myopia and weak willpower, this index is always less than or equal to 1.

Creating twin-models with public and local learning, our paper sheds new light on the foregoing problems. Our *analytical* model with public information used by the shortsighted workers gives relatively simple results, especially for the dependence of the steady state on the matching rate and the relative propensity to save. Our *agent-based* model incorporates more realistic, local learning, where the more shortsighted workers also learn from less shortsighted ones.[1]

The proponents of voluntary systems justify the subsidies as follows: a mandatory system does not and cannot ensure high enough pensions, and the mostly shortsighted workers must be made interested in raising their old-age incomes through a voluntary system (e.g. Poterba et al. 1996). The opponents are afraid that these subsidies are poorly targeted, mostly subsidize the well-paid savers, while worsening the burden of the others by generating *tax expenditures* (Engen et al. 1996; Duflo et al. 2007). Hubbard and Skinner (1996) tried to synthesize both approaches, while OECD (2005) and Hinz et al. (2013) summarized the practice of various countries. Up to now the foregoing tax expenditures have generally been quite low though nonnegligible (about 0.7% of the GDP in the US), but in the case of a possible contraction of the mandatory system they may become much higher.

Since Modigliani and Brumberg (1954) and Samuelson (1958), models of life-cycle saving and of overlapping generations have been extensively studied, respectively. A new era started with Auerbach and Kotlikoff (1987) which generalized the partial equilibrium framework into a general equilibrium one: not only savings depend on the interest rates but the interest rates also depend on savings through accumulated capital.

[1] In a previous version, Király and Simonovits (2016) used the expression *global learning* for learning by using public information.

By replacing traditional life-cycle savings with mandatory and voluntary pensions, these models have become more realistic.[2]

A common problem with these models, however, is that they assume that the individuals have an extraordinary sophistication to solve the corresponding parts of their optimization problem and the willpower to achieve the results. It is widely documented, however, that a large share of the population have quite limited cognitive abilities (for a survey, see Lusardi and Mitchell 2014), quite limited information (Barr and Diamond 2008, Box 4.2) and weak willpower.

A class of very simple life-cycle models operate with given interest rates and wages (small open economy). In such models, various workers' ordinary life-cycle saving processes are independent, but adding government matching via a voluntary pension system introduces interdependence. Indeed, even if somebody does not participate in the scheme, he pays taxes according to the same earmarked tax rate. This may be the reason why in voluntary systems, individual optimization is mathematically quite difficult (see Appendix B in Király and Simonovits 2016) even if in addition to heterogeneously myopic workers, only two working age periods are distinguished. Only by neglecting the difference between young and old workers was Simonovits (2011) able to obtain analytical results on the impact of the matching rate and of the cap on income redistribution in such a transfer system. A more realistic approach to life-cycle savings is based on *behavioral economics* [started by Thaler and Benartzi (2004) and crowned by a recent survey by Chetty (2015)] but they neglect tax expenditures.

We start the discussion of learning to save in an analytical model. For simplicity, we assume a stationary population without growth, inflation and interest. To make the impact of the individual decisions on the macro state negligible, we assume that there are a continuum of workers. Following Feldstein (1985), we distinguish at least farsighted and shortsighted workers. After the steady-state analysis, we assume that the government unexpectedly introduces a matching scheme in period 0, and this initiates a new behavior for both types. The appearance of government matching and its tax financing make the shortsighted workers aware that the farsighted save for the future and even its size can be guessed. The inadequacy of the shortsighted workers' savings is measured by the standard deviation in their life-cycle consumption, while the redistribution from the shortsighted to the farsighted workers is measured by the standard deviation of the population lifetime average consumptions. For the sake of brevity, the two standard deviations are distinguished by adjectives *internal* and *external*.

To simplify the calculations, we assume that the farsighted workers try to smooth their consumption paths without any intertemporal substitutability. Our analytical results on life-cycle saving are as follows: (a) By saving in a tax-favored system, the farsighted workers simply exploit the shortsighted ones. (b) We assume a special and admittedly artificial form of learning from public information: the active shortsighted workers guess the amount of their farsighted counterparts' saving as the ratio

[2] For example, in a calibrated general equilibrium model of the German economy, Fehr et al. (2008) showed that if the voluntary pillar is extended, then existing generations lose and future generations gain. In addition, the assumption of rational expectations makes the foregoing models extremely complex (for an alternative with naive expectations, see Molnár and Simonovits 1998).

of the tax rate and the matching rate provided by *government statistics*, and due to shortsightedness (and weak willpower), they save only a given share of this estimation: the share to be called *relative propensity to save* [see (6) below for its definition]. Note that if the share of the farsighted workers is very low, then they only play the role of the catalyzer but without them the model ceases to work. The process converges from the old to a new steady state (at least for moderate matching rates) and the degree of exploitation is significantly reduced. (c) We can simply disaggregate the aggregate behavior of shortsighted workers according to their different relative saving propensities and obtain subtypes.

In the second model, we assume that these shortsighted subtypes also learn *locally* from each other. By adding local learning to using public information, we study an *agent-based model* (for short, ABM). These models generally enhance the realism of economic modeling (see e.g. Tesfatsion 2006). The main innovation of ABMs is that by sacrificing the ability to derive analytical results, they are able to describe more realistically the behavior of interacting heterogeneous agents. This methodology has been successfully used in several fields of economics. For example, the topic of tax evasion, related to our problem, was investigated—among others—in Méder et al. (2012), Pickhard and Prinz (2013) and Bertotti and Modanese (2016).

Quite recently, Varga and Vincze (2017) used an ABM to analyze a very abstract model of ordinary saving. They assumed a very long (practically infinite) horizon and excluded mandatory as well as voluntary pensions. They distinguished three types of agents: buffer-stock savers (who follow the prescriptions of the life-cycle model, smooth their consumption path by saving), permanent income savers (forward looking individuals without prudence) and myopic savers (who spend most of their disposable income on current consumption). The main message of that paper is that notwithstanding permanent learning, different types can coexist for a very long time.

Applying the ABM approach to *life-cycle savings*, especially to voluntary pension looks promising. Already Duflo and Saez (2003) emphasized the influence of colleagues' choices on participation in voluntary pension plans. Here we try to explain an empirically verified fact: though the share and the extent of participation in tax favored systems are increasing functions of the wages; even controlling for wages, both indicators are heterogeneous (Baily and Kirkegaard 2009, Table 8.1, p. 456). We take homogeneous wages, neglect the cap on the voluntary contributions, thereby eliminate unmatched savings above the matched savings.

We highlight the following ABM-results: (i) in the basic run, some heterogeneity in savings of the shortsighted workers remains; (ii) increasing the spread between the propensities diminishes both standard deviations; (iii) the increase in the number of types diminishes the external standard deviation but increases the internal standard deviation; (iv) randomly perturbing the network may homogenize the shortsighted workers' savings; (v) the rise in the number of acquaintances does not reduce the standard deviations; (vi) diminishing the density of the connections by a factor of 4, the convergence is much slower; (vii) even if the workers revise their strategy annually rather than per decade, the welfare is not raised; and (viii) raising the relative propensities, the savings increase. In summary: the behavior of the complex system (ABM) cannot be fully understood from its simplified version (analytical model).

Table 1 Certain properties of selected models

Models	Dynamic saving	Tax expenditure	Simple rules	Local learning
Choi et al. (2004)	+	–	+	–
Fehr et al. (2008)	+	+	–	–
Simonovits (2011)	–	+	–	–
Analytical model	+	+	+	–
ABM	+	+	+	+

Table 2 Features of pension systems of selected countries

Country	Mandatory		Voluntary	
	Progressive	Size	Progressive	Size
United States	Medium	Medium	No	Medium
Germany	Weak	Large	Medium	Small
Netherlands	Strong	Large	No	Medium
Czech Republic	Strong	Large	Strong	Small
Hungary	Weak	Large	No	Large
Twin-models	No	Medium	No	Large

Further work is needed to check the robustness of these results especially the details of local learning.

To place the current paper in the related literature, Table 1 compares the presence of the following properties of five selected models of voluntary pension systems: dynamic saving, tax expenditure, simple saving rules and local learning (+ means yes, – means no). The selected models are Choi et al. (2004), Fehr et al. (2008), Simonovits (2011) and the twin-models of the current paper: analytical versus ABM. [Note that Simonovits (2011) is also an analytical model with such a simple structure that has no room for learning.] The analytical model has three and the ABM has four +s, while Simonovits (2011) has only one.

Table 2 shows how the core of our twin-models is related to five selected countries' pension systems according to the strength of progressivity and the size of the mandatory (public + private) and of the voluntary systems, respectively. We see that our twin-models go even beyond the German and the Hungarian systems in eliminating any redistribution in the mandatory system, and it copies the US system's medium size. Concerning the voluntary system, our models are similar to the US, the Dutch and the Hungarian systems having no progressivity. It is not shown in the table, but our twin-models resemble the German voluntary system in having mandatory life annuities, and approximates the Hungarian system with its very high cap on the voluntary savings. In summary, we have copied various features of various countries arbitrarily, just to make the twin-models as simple as possible, to focus on the learning dimension.

Though our model family admittedly lies quite far from any real voluntary pension system, we formulate some policy suggestions. (i) Models of voluntary pension systems should take into account the tax expenditure of operating the voluntary pension system. (ii) The design of the voluntary system should be in harmony with that of the mandatory system: probably a progressive voluntary system fits a proportional mandatory one and a proportional voluntary system fits a progressive mandatory one. (iii) If the government wants to strengthen the voluntary savings of the shortsighted, it should increase the matching rate and decrease the cap (not discussed here).

The structure of the remainder of the present paper is as follows: Sect. 2 discusses an analytical model of life-cycle saving, where the shortsighted workers learn only from public information. Section 3 studies the corresponding ABM. Section 4 concludes.

2 An analytical model

In this section we study an analytical model with learning from public information. As a starter, Sect. 2.1 assumes passive myopic workers who do not learn and do not save at all. Section 2.2 activates them by introducing the relative propensity to save but the model is kept static. Section 2.3 introduces dynamics into the saving model.

2.1 Passive shortsighted workers

We shall consider a simple model of mandatory and voluntary pensions. To simplify exposition, we consider a stationary population, with overlapping cohorts. Every time period, D cohorts live together and at the end of the period (not modeled until the next subsection), every cohort becomes older by one period except for the oldest, which dies and the youngest, which just enters the labor market. There are $R > 0$ working cohorts and $D - R > 0$ retired cohorts, where R and D are positive integers. The workers earn unitary wages, pay $\tau > 0$ as a mandatory pension contribution. The retired cohorts receive universal pension benefits b. Introducing notations for the ratios of working span to total adult life span and that of working span to retirement span,

$$\rho = \frac{R}{D} < 1 \quad \text{and} \quad \beta = \frac{\rho}{1-\rho},$$

the benefit is

$$b = \frac{R\tau}{D-R} = \frac{\rho\tau}{1-\rho} = \beta\tau. \tag{1}$$

It is easy to see that the net wage $1-\tau$ and the pension benefit b are equal (consumption smoothing) if the contribution rate is equal to

$$\bar{\tau} = 1 - \rho.$$

If the contribution rate is high, then workers restrain their labor supply and unreport a large part of their wage. Therefore the government keeps the contribution rate well below this *maximal* value: $\tau < \bar{\tau}$ and encourages private savings in a voluntary pension system: the per-period saving is denoted by $s \geq 0$. To promote participation, every euro paid into the voluntary system is matched by $\alpha > 0$ euros by the government. In contrast to the bulk of the literature, we explicitly model the earmarked tax needed to finance such matching from wage taxes with a flat rate θ. There is no other tax in our models. A basic observation is that different types of workers—even with identical earnings—save different amounts in voluntary systems. We could generate such a behavior by assuming heterogeneous discount factors (cf. Simonovits 2011) but we rely on simpler methods.

Until the end of this Subsection, we shall assume that there are only two (ageless) types: shortsighted (L, he) and farsighted (H, she), with shares $f_L, f_H > 0$ and $f_L + f_H = 1$. In this Subsection, the shortsighted worker is passive, does not save at all: $s^L = 0$ and the farsighted worker saves s^H to smooth her consumption path. Denoting worker i's and pensioner i's ($i = L, H$) age-invariant consumption per period by $c_{1,i}$ and $c_{D,i}$ (alluding to the start of the working stage, 1 and the end of the retirement stage, D), respectively, we have the following tax equation:

$$\theta = \alpha f_H s^H \tag{2}$$

and consumption equations:

$$c_{1,L} = 1 - \tau - \theta, \qquad c_{D,L} = b \tag{3-L}$$

and

$$c_{1,H} = 1 - \tau - \theta - s^H, \qquad c_{D,H} = \beta[\tau + (1+\alpha)s^H]. \tag{3-H}$$

We assume that H saves as much as needed to smooth out her projected consumption path:

$$c_{1,H} = c_{D,H} = c^H, \quad \text{i.e.} \;\; 1 - \tau - \theta - s^H = \beta[\tau + (1+\alpha)s^H].$$

We have then

$$s^H = \frac{\chi - \theta}{1 + \beta(1+\alpha)}, \quad \text{where} \quad \chi = 1 - (1+\beta)\tau > 0. \tag{4}$$

We display the special value of s^H at $\alpha = 0$, the second part of the equation shows how mandatory pension contributions crowd out savings:

$$s^H(0) = \frac{\chi}{1+\beta} = \frac{1}{1+\beta} - \tau. \tag{4'}$$

Substituting (4) into (2) yields an implicit equation for the balanced tax rate:

$$\theta = \alpha f_H \frac{\chi - \theta}{1 + \beta(1+\alpha)}.$$

Hence follows

Theorem 1 *In the two-type analytical model with passive shortsighted workers, the government sets the balanced tax rate*

$$\theta^{\mathrm{o}} = \frac{\alpha f_H \chi}{1 + \beta(1+\alpha) + \alpha f_H} > 0. \tag{5}$$

Then every farsighted worker chooses her saving $s^H = \theta^{\mathrm{o}}/(\alpha f_H)$ and every shortsighted worker saves nothing.

Remark 1. In this model, the introduction of voluntary pension saving simply redistributes from the shortsighted to the farsighted workers. The higher the matching rate, the stronger the redistribution.
2. In such a zero-sum game, the use of voluntary pensions is only justifiable if there is wage heterogeneity ($w_L < w_H$) and the mandatory (public) pension is progressive: $b_i = \beta_0 + \beta w_i$, with $\beta_0 > 0$ and $\beta > 0$, but this is beyond the scope of this paper.

To display perverse redistribution, we also determine the shortsighted workers' *lifetime average* consumption $c^L = \rho c_1^L + (1-\rho)c_D^L$. As the matching rate increases, so decreases L's lifetime average consumption. Using the obvious formula for the expected average consumption: $c = f_L c^L + f_H c^H = \rho$, the simplest measure of perverse redistribution is the *external standard deviation* of the lifetime average consumptions of the whole population:

$$\varepsilon_E = \left[f_L(c^L - \rho)^2 + f_H(c^H - \rho)^2 \right]^{1/2}.$$

In addition, to measure the *internal standard deviation*, we also introduce

$$\varepsilon_I = \left[f_L \rho (c_1^L - c^L)^2 + f_L(1-\rho)(c_D^L - c^L)^2 \right]^{1/2}.$$

To help understanding, we shall numerically illustrate our results. Let us calculate in decades (bold symbols refer to decades rather than years): $\mathbf{R} = 4, \mathbf{D} = 6$, $\rho = 2/3$ and choose a contribution rate $\tau = 0.2$ far below the maximum: $\bar{\tau} = 1/3$. Table 3 displays the two types' characteristics for three matching rates: $\alpha = 0, 0.5, 1$; for population shares $f_L = 3/4$ and $f_H = 1/4$. As the matching rate increases, so decreases L's lifetime average consumption: at $\alpha = 1$, the earmarked tax rate is equal to 0.019 and the average consumption of the shortsighted type drops from 0.667 to 0.654. As the matching rate grows, the internal standard deviation diminishes from 0.163 to 0.156, while the external standard deviation grows from zero to 0.022. To relate these values to the extreme standard deviations where everybody is shortsighted and there is no mandatory pension, we give the corresponding maximum and minimum, respectively: $\bar{\varepsilon}_I = 0.47$ and $\bar{\varepsilon}_E = 0$.

Table 3 Worker and pensioner consumption: passive L, varying matching

Matching rate	Tax rate	Consumption of				SD	
		H-type	L- worker	L-pensioner	L-average	Internal	External
α	θ	c^H	c_1^L	c_D^L	c^L	ε_I	ε_E
0.0	0	0.667	0.800	0.4	0.667	0.163	0
0.5	0.012	0.691	0.788	0.4	0.659	0.159	0.014
1.0	0.019	0.705	0.781	0.4	0.654	0.156	0.022

SD standard deviation

2.2 Active workers: steady state

In Sect. 2.1, we assumed that shortsighted workers are passive, they do not understand anything from the logic of the system, they simply pay their dues without having any return. From now on we assume that these workers are active, they understand something and react to exploitation by saving. In Sect. 2.2 we rely on steady state analysis, and in Sect. 2.3 we turn to the dynamics.

Every worker of type L presumes that all the other workers (including Ls) are type H and knowing the tax rate θ and the matching rate α, relying on (2), he naively underestimates their per-capita saving to be equal to θ/α. Due to his myopia and weak willpower, he is ready to save only γ times this quantity, $(0 < \gamma \leq 1)$, therefore

$$s^L = \frac{\gamma\theta}{\alpha}, \qquad \alpha > 0. \tag{6}$$

We shall refer to γ as *relative propensity to save*. Retaining (4), the modified tax balance equation (2) becomes

$$\theta = \gamma f_L \theta + \alpha f_H \frac{\chi - \theta}{1 + \beta(1 + \alpha)}. \tag{2'}$$

With a simple calculation, we have obtained the government tax rate and the two types' saving rates.

Theorem 2 *The steady state with active shortsighted workers is characterized by*

$$\theta_\gamma^{\text{o}} = \frac{\alpha f_H \chi}{\nu}, \qquad s^H = \frac{(1 - \gamma f_L)\chi}{\nu} \geq s^L = \frac{\gamma f_H \chi}{\nu}, \tag{7}$$

where

$$\nu = (1 - \gamma f_L)[1 + \beta(1 + \alpha)] + \alpha f_H > 0.$$

Remark 1. Looking at the steady state balanced tax rate (7) with active workers, note that the higher the relative propensity to save γ, the higher the balanced tax rate, and the lower the redistribution. For $\gamma = 1$, the shortsighted become farsighted

Table 4 Worker and pensioner consumption: active L, varying propensities

Relative propensity to save	Tax rate	Consumption of				SD	
		H-type	L-worker	L-pensioner	L-average	Internal	External
γ	θ	c^H	c_1^L	c_D^L	c^L	ε_I	ε_E
0.00	0.019	0.705	0.781	0.400	0.654	0.156	0.022
0.25	0.023	0.701	0.771	0.423	0.655	0.142	0.020
0.50	0.030	0.696	0.756	0.459	0.657	0.121	0.017
0.75	0.041	0.687	0.728	0.523	0.660	0.084	0.012
1.00	0.067	0.667	0.667	0.667	0.667	0	0

SD standard deviation, $\alpha = 1$

and exploitation disappears. Note that the tax rate with passive workers is just the product of the share of farsighted workers and the tax rate with maximal $\gamma = 1$: $\theta_0^o = f_H\theta_1^o$.

2. Disaggregating the shortsighted workers into $n - 1 > 1$ types with different γ_is, we can open the door to multitype models (to be studied in Sect. 3). Indeed, let $f_1, f_2, \ldots, f_{n-1}$ be the population share of the shortsighted workers with relative saving propensities $\gamma_1 < \gamma_2 < \cdots < \gamma_{n-1} \le 1$, respectively. Then the disaggregated model can be aggregated as

$$f_L = \sum_{i=1}^{n-1} f_i < 1 \quad \text{and} \quad \gamma = \frac{\sum_{i=1}^{n-1} f_i\gamma_i}{f_L} < 1.$$

Then s^L in Theorem 2 can also be disaggregated:

$$s^i = \frac{\gamma_i f_H \chi}{\nu}, \qquad i = 1, \ldots, n-1. \tag{7M}$$

Returning to the two-type model, Table 4 displays the impact of the relative propensity to save γ with $\alpha = 1$. The first row replicates the third row of Table 3. As γ increases from 0 to 1, the earmarked tax rate rises from 0.019 to 0.067, and even the shortsighted type's consumption path becomes smooth, i.e. age-invariant. Eventually both the internal and the external standard deviations drop to zero.

2.3 Dynamic analytical model

In a standard overlapping generations model, the agents differ not only in age but also in the time they start working. In our dynamic analytical model, we shall denote the age of workers by $a = 1, 2, \ldots, R$ and of pensioners by $a = R + 1, \ldots, D$. Every period t, D adult cohorts overlap: those entering the labor market in period $t, t - 1, \ldots, t - D + 1$, respectively. For technical reasons we assume that the new cohort entered the labor market in period $t + 1$ rather than t. The subindex triple $(a, i, t + a)$ refers to type i of age a in period $t + a$. To have a recursive model, we assume that in every period $t \geq 0$, the government determines and announces the actual matching rate $\alpha > 0$, the appropriate tax rate θ_t and then the various types calculate the corresponding age-dependent savings. First we determine the longitudinal saving paths and then transform them into cross-sectional saving profiles.

Longitudinal equations

For each pair (a, i), the end-of-period financial assets (accumulated savings including matching) satisfy a dynamic relation:

$$S_{a,i,t+a} = S_{a-1,i,t+a-1} + (1 + \alpha)s_{a,i,t+a}, \qquad t = 0, 1, 2, \ldots, \tag{8}$$

where the initial conditions are given:

$$S_{a-1,i,-1}, \qquad a = 2, \ldots, R \quad \text{and} \quad i = L, H.$$

It is logical but not necessary to assume that these initial states are consistent with optimal savings without matching.

Let E_t be the tax expenditure in period t per capita. We have the following identity:

$$E_t = \alpha \sum_{i=L}^{H} f_i \sum_{a=1}^{R} s_{a,i,t}. \tag{9}$$

If the government were supposed to cover these expenditures every period from its revenue $\theta_t R$, then it should solve a complex fixed-point problem because through $s_{a,i,t}$s, E_t depends on θ_t. Instead, we relax the previous tax equation ($2'$) and allow the government to run temporary surpluses and deficits, financed by the external world, resulting in the per-capita stock of government debt at the end of period t:

$$\mathcal{D}_t = \mathcal{D}_{t-1} + E_t - R\theta_t, \qquad t = 0, 1, 2, \ldots, \qquad \mathcal{D}_{-1} = 0. \tag{10}$$

Using a trial-and-error method, at the beginning of period $t > 0$ the government chooses and announces the tax rate which would have covered the expenditures in the previous period:

$$\theta_t = \frac{E_{t-1}}{R}. \tag{11}$$

Note that this leads to $\mathcal{D}_t = \mathcal{D}_{t-1} + E_t - E_{t-1} = E_t$. (At the end of this Subsection, (11) is replaced by ($11'$) which stabilizes the debt at zero.)

By definition, we have two classes of consumption equations.

Consumption at work:

$$c_{a,i,t+a} = 1 - \tau - \theta_{t+a} - s_{a,i,t+a}, \qquad a = 1, 2, \ldots, R. \tag{12a}$$

Consumption at retirement:

$$c_{a,i,t+R+1} = \cdots = c_{a,i,t+D} = b + d_{i,t+R}, \qquad a = R+1, \ldots, D, \tag{12b}$$

where the private life annuity is given by

$$d_{i,t+R} = \psi S_{R,i,t+R}, \quad \text{where} \quad \psi = \frac{1}{D-R}, \qquad i = L, H. \tag{13}$$

Instead of the steady state estimation of s_L [(7)], assume that the age- and time-dependent saving varies with the time-variant tax rate θ_{t+a}:

$$s_{a,L,t+a} = \frac{\gamma \theta_{t+a}}{\alpha}, \qquad a = 1, \ldots, R. \tag{14}$$

In our dynamic analytic model, even the farsighted workers do not know their future savings, they naively assume that they will save the same amount until retiring as they save now. Otherwise, they would have to solve the whole model for themselves, which would be an excessive requirement.
Projected private life-annuity at age a:

$$d_{a,H,t+R} = \psi[S_{a-1,H,t+a-1} + (1+\alpha)(R-a+1)s_{a,H,t+a}]. \tag{15}$$

Projected consumption at retirement:

$$\tilde{c}_{a,H,t+R+1} = \cdots = \tilde{c}_{a,H,t+D} = b + d_{a,H,t+R}. \tag{16}$$

While working, type H always tries to smooth her future consumption path, $c_{a,H,t+a} = \tilde{c}_{a,H,t+R+1}$, i.e. by (15)–(16):

$$1 - \tau - \theta_{t+a} - s_{a,H,t+a} = b + \psi S_{a-1,H,t+a-1} + \psi(R-a+1)(1+\alpha)s_{a,H,t+a},$$

hence her 'current' age- and time-dependent saving is given by

$$s_{a,H,t+a} = \frac{\chi - \psi S_{a-1,H,t+a-1} - \theta_{t+a}}{1 + \psi(R-a+1)(1+\alpha)} = \varphi_a(\chi - \theta_{t+a}) - \sigma_a S_{a-1,H,t+a-1}, \tag{17}$$

where

$$\varphi_a = \frac{1}{1+\psi(R-a+1)(1+\alpha)} \quad \text{and} \quad \sigma_a = \psi\varphi_a.$$

Cross-sectional equations
To use (9)–(11), we shall need the saving rules in t rather than in $t+a$, therefore we shift (14), (17), (9) and (8) back by a.

L-saving:

$$s_{a,L,t} = \frac{\gamma\theta_t}{\alpha}, \qquad a = 1, \ldots, R. \tag{14'}$$

H-saving:

$$s_{a,H,t} = \varphi_a(\chi - \theta_t) - \sigma_a S_{a-1,H,t-1}, \qquad a = 1, \ldots, R. \tag{17'}$$

Tax expenditure:

$$E_{t-1} = \alpha \sum_{i=L}^{H} f_i \sum_{a=1}^{R} s_{a,i,t-1}. \tag{9'}$$

Type-specific financial assets:

$$S_{a,i,t} = S_{a-1,i,t-1} + (1+\alpha)s_{a,i,t}, \qquad a = 1, 2, \ldots, R, \qquad i = L, H, \tag{8'}$$

where the initial conditions are given:

$$\theta_{-1} = 0, \quad S_{a-1,i,-1}, \qquad a = 1, \ldots, R, \qquad i = L, H.$$

Furthermore, the matching rate is zero before 0 and is a positive constant after –1.

To minimize the dimension of the system, we drop the debt dynamics as a reducible component, and $(S_{a-1,L,-1})_{a=2}^{R}$ as reducible initial conditions. Substituting (14′) into (9′–11′) and repeating the remaining equations of the irreducible system, namely (17′) and (8′) for $t = 1, 2, \ldots,$ and $\alpha \neq 0$:

$$\theta_t = \gamma f_L \theta_{t-1} + \alpha R^{-1} f_H \sum_{a=1}^{R} s_{a,H,t-1}, \tag{18}$$

$$s_{a,H,t} = \varphi_a \chi - \varphi_a \gamma f_L \theta_{t-1} - \varphi_a R^{-1} \alpha f_H \sum_{x=1}^{R} s_{x,H,t-1} - \sigma_a S_{a-1,H,t-1}, \tag{19}$$

and (8′). For example, during the transition, where $0 \leq t < a-1$, $a = 2, \ldots, R$, (8′) takes the form

$$S_{a,i,t} = s_{1,i,t-a+1} + \cdots + s_{a-t-1,i,-1} + (1+\alpha)(s_{a-t,i,0} + \cdots + s_{a,i,t}), \quad i = H, L.$$

System (18)–(19)–(8′H) is an inhomogeneous linear system of dimension $m = 2R - 1$.

Theorem 3 *(a) In the two-type dynamic analytical model, the government sets the tax rate θ_t according to (18), the farsighted and the active shortsighted workers save according to (19) and (14′), respectively.*

(b) *For any sufficiently low matching rate* α*, the system converges to the new steady state of Theorem 2.*

Remark 1. It is implicitly assumed that the initial values are equal or sufficiently close to their old steady state values to generate viable paths, i.e. $s_{a,i,t+a}, c_{a,i,t+a} \geq 0$ for all $(a, i, t+a)$s.

2. It is an open question how low the matching rate should be to guarantee the stability of the new steady state in general, but Example 1 below provides a special answer.
3. The model can easily be generalized for heterogeneous shortsighted workers with different γ_is as in Theorem 2.

Proof (a) We have proved part (a) above.

(b) To prove stability, we can drop the constant terms from (19). Then we have a simple solution for $\alpha = 0 = \theta_t$:

$$s_{a,H,t} = -\sigma_a S_{a-1,H,t-1}. \tag{19'}$$

Substituting (19′) into (8′) results in

$$S_{a,H,t} = (1-\sigma_a)S_{a-1,H,t-1} = (1-\sigma_a)\cdots(1-\sigma_1)S_{0,H,t-a} = 0 \quad (t \geq a). \tag{20}$$

By continuity, stability survives for sufficiently low matching rates. □

To obtain a clear picture of this complex dynamics, we consider the simplest case, OLG 1-1.

Example 1 Let $\mathbf{R} = 1$ and $\mathbf{D} = 2$. In this case, neglecting the transition, $S_{1,H,t} = (1+\alpha)s_{1,H,t}$:

$$\theta_t = \gamma f_L \theta_{t-1} + \alpha f_H s_{1,H,t-1} \tag{18''}$$

and

$$s_{1,H,t} = \frac{\chi - \theta_t}{2+\alpha}. \tag{17''}$$

Shifting (17″) back by 1 period and inserting the shifted (17″) into (18″) yields

$$\theta_t = \alpha f_H \frac{\chi}{2+\alpha} + \left[\gamma f_L - \frac{\alpha f_H}{2+\alpha}\right]\theta_{t-1}.$$

The path generated by this first-order linear difference equation is obviously stable. For $\gamma^* = \dfrac{\alpha f_H}{(2+\alpha) f_L}$, the tax rate jumps to the steady state in $t = 2$. For $0 < \gamma < \gamma^*$, the tax rate oscillates around the steady state, while for $\gamma^* < \gamma \leq 1$, the tax rate increasingly converges to the steady state. (Note that the second interval is empty, i.e. $\gamma^* > 1$ if and only if $(2+\alpha)/[2(1+\alpha)] < f_H \leq 1$.)

Table 5 H-saving profiles in overlapping generations

Period	Debt	Tax rate	SD		Saving of H-workers			
			Internal	External	Youngest	Younger	Older	Oldests
t	$\mathcal{D}_{\mathbf{t}}$	$\theta_{\mathbf{t}}$	$\varepsilon_{I,\mathbf{t}}$	$\varepsilon_{E,\mathbf{t}}$	$s_{\mathbf{1},H,\mathbf{t}}$	$s_{\mathbf{2},H,\mathbf{t}}$	$s_{\mathbf{3},H,\mathbf{t}}$	$s_{\mathbf{4},H,\mathbf{t}}$
0	0.088	0.000	0.232	0.013	0.080	0.083	0.089	0.100
1	0.109	0.022	0.212	0.025	0.076	0.074	0.076	0.078
2	0.118	0.027	0.201	0.020	0.075	0.074	0.077	0.082
3	0.120	0.029	0.191	0.019	0.074	0.074	0.077	0.080
4	0.121	0.030	0.182	0.017	0.074	0.074	0.076	0.080
5	0.122	0.030	0.173	0.015	0.074	0.074	0.076	0.080
6	0.122	0.030	0.171	0.014	0.074	0.074	0.076	0.080

SD standard deviation

Finally, Tables 5, 6, 7 display the numerical illustrations for $\mathbf{R} = 4$ and $\mathbf{D} = 6$ (decades), matching rate $\alpha = 1$ and the relative propensity to save $\gamma = 1/2$ (Table 4, middle row). We expect that the process converges to the corresponding steady state. Our expectations are correct, at least for the initial values $\theta_0 = 0$, $\mathcal{D}_{-1} = 0$. Furthermore we choose the initial values for L and H savings as 0 and $s^H(0)$ [see (4')], plus the accumulated savings, belonging to $\alpha = 0$. (We have experimented with other initial states and we obtained qualitatively the same results.)

Table 5 displays the paths of the debt, of the tax rate and of the H-saving. As expected, the debt converges to 0.12 while the tax rate converges to the steady state 0.03. When the transfer system is unexpectedly introduced in period $t = 0$, the farsighted but still naive workers' savings drop (and their consumption jumps), at least temporarily. As the matching system builds up, the H-savings drop to the steady state values of $s^H = 0.074$, regardless of age. The internal and external standard deviations converge to their respective steady state values. (We have repeated the calculations for a number of other combinations of parameter values and obtained stability.)

Table 6 presents the farsighted workers' consumption paths. The lifetime average consumption is also displayed though not for a longitudinal path but for a cross-section profile: $c_t^i = D^{-1} \sum_{a=1}^{D} c_{a,i,t}$, $i = H, L$.

In the shortsighted workers' consumption paths (Table 7), similar overconsumption can be observed which stabilizes quite fast at 0.756, while the pensioners' consumption rises from 0.4 to 0.459. Comparing the two life-consumptions at $\mathbf{t} = 6$, we see the difference: $c_6^L = 0.656 < 0.689 = c_6^H$.

At the end of the Section, we make three short remarks.

1. Repeating the calculations for various matching rates (α), frequencies of myopes (f_L) and relative propensities to save (γ), the stability remains valid.
2. It is evident that calculating the tax rate, adding a given part of the past debt to the past expenditure in (11), will asymptotically eliminate the debt. Formally,

Table 6 H-consumption profiles in overlapping generations

Period	Consumption of					
	H-workers				H	
	Youngest	Younger	Older	Oldest	Pensioner	Average
t	$c_{\mathbf{1},H,\mathbf{t}}$	$c_{\mathbf{2},H,\mathbf{t}}$	$c_{\mathbf{3},H,\mathbf{t}}$	$c_{\mathbf{4},H,\mathbf{t}}$	$c_{\mathbf{5},H,\mathbf{t}}$	$c_{\mathbf{t}}^{H}$
0	0.720	0.717	0.711	0.700	0.667	0.697
1	0.702	0.703	0.702	0.700	0.700	0.701
2	0.698	0.698	0.696	0.691	0.678	0.690
3	0.696	0.697	0.694	0.691	0.682	0.690
4	0.696	0.696	0.693	0.690	0.680	0.689
5	0.696	0.696	0.693	0.690	0.680	0.689
6	0.696	0.696	0.693	0.690	0.680	0.689

$c_{\mathbf{6},H,\mathbf{t}} = c_{\mathbf{5},H,\mathbf{t}-1}$

Table 7 L-consumption profiles in overlapping generations

Period	Consumption of					
	L-workers				L	
	Youngest	Younger	Older	Oldest	Pensioner	Average
t	$c_{\mathbf{1},L,\mathbf{t}}$	$c_{\mathbf{2},L,\mathbf{t}}$	$c_{\mathbf{3},L,\mathbf{t}}$	$c_{\mathbf{4},L,\mathbf{t}}$	$c_{\mathbf{5},L,\mathbf{t}}$	$c_{\mathbf{t}}^{L}$
0	0.800	0.800	0.800	0.800	0.400	0.667
1	0.767	0.767	0.767	0.767	0.400	0.645
2	0.759	0.759	0.759	0.759	0.411	0.643
3	0.756	0.756	0.756	0.756	0.425	0.645
4	0.755	0.755	0.755	0.755	0.439	0.650
5	0.755	0.755	0.755	0.755	0.454	0.654
6	0.754	0.754	0.754	0.754	0.459	0.656

$c_{\mathbf{6},L,\mathbf{t}} = c_{\mathbf{5},L,\mathbf{t}-1}$

$$\theta_t = \frac{E_{t-1}}{R} + \zeta \mathcal{D}_{t-1}, \tag{11'}$$

where $\zeta > 0$ is an adjustment coefficient, for example, $\zeta = 1/R$. Then (18) modifies to

$$\theta_t = \gamma f_L \theta_{t-1} + \alpha R^{-1} f_H \sum_{a=1}^{R} s_{a,H,t-1} + \zeta \mathcal{D}_{t-1}.$$

3. A referee noted that making γ age-dependent, furthermore, increasing with age in (14), the model would be more realistic. For example, choosing an initial and a final γ, $\gamma_1 \geq 0$ and $\gamma_R \leq 1$, with $\gamma_1 < \gamma_R$; and connecting them with $\gamma_a = [(R-a+1)\gamma_1 + a\gamma_R]/R$ would do the job. But then (18) should also be modified.

3 An agent-based model

In the previous model, even the active shortsighted workers learn relatively little. An important feature of the ABM is that everybody learns from others, therefore we shall apply ABM to make shortsighted workers learn to save in a voluntary pension system not only by using public information but also local information. The basic idea of local learning is as follows: every period, each shortsighted worker with a given age and endogenously changing (and improving) type looks around among his acquaintances (older by one period than he is) and chooses that type which promises the highest lifetime utility. We shall first formulate the model, then present numerical illustrations.

3.1 Theoretical analysis

We assume that there are n types, where $n > 2$ is a relatively small integer. Type i is characterized by its relative propensity to save $\gamma_i = i/n$, $i = 1, 2, \ldots n-1$, with population shares $f_i > 0$, $\sum_{i=1}^{n-1} f_i = f_L < 1$ and average relative propensity to save $\gamma = f_L^{-1} \sum_{i=1}^{n-1} f_i \gamma_i$. Type n with frequency $f_H = 1 - f_L$ is farsighted. (Without assuming the existence of farsighted agents, the steady state tax rate would remain zero forever.) We assume that there are a finite but large number of workers ($M = RN$). Therefore, when they start to work, the number of type i workers aged 1 is Nf_i in each cohort, indexed by $k = N_{i-1} + 1, \ldots, N_i$, where $N_{i+1} = N_i + Nf_i$, $N_0 = 0$.

First we modify the per-capita tax expenditure formula (9) as

$$E_t = \alpha \sum_{i=1}^{n} f_i \sum_{k=N_{i-1}+1}^{N_i} \sum_{a=1}^{R} s_{a,k,t}, \qquad t = 0, 1, 2, \ldots. \tag{25}$$

Király and Simonovits (2016, pp. 18–19) also applied the lifetime utility function, which is maximized in standard economics. In our context its use would contradict the bounded rationality of the shortsighted workers. Rather we check a simpler indicator, the lifetime average consumption mentioned in Sect. 2. In our new setting, its projected value at $(a, k, t + a)$ is given by the weighted average cumulated consumption until age a $C_{a-1,t+a-1}$ plus the current consumption $c_{a,k,t+a}$ and the projected future old-age consumption $\tilde{c}_{a,k,R+1}$:

$$c_{a,t+a}^{k} = \frac{1}{D}[C_{a-1,k,t+a-1} + (R - a + 1)c_{a,k,t+a} + (D - R)\tilde{c}_{a,k,R+1}], \tag{26}$$

where

$$C_{a,k,t+a} = \sum_{x=1}^{a} c_{x,k,t+x} = C_{a-1,t+a-1} + c_{a,k,t+a}.$$

Again, by (26), (12) and (15)–(16), in period $t+a$, $c_{a,t+a}^{k}$ is a simple linear function of a single variable $s_{a,k,t+a}$. Finally, at retirement, the projected value is crystallized into

$c_t^k = c_{R,t+R}^k$. This indicator is far from ideal, because it does not reflect a main aim of the pension system: consumption smoothing. Nevertheless, in the world of bounded rationality, together with ε_I and ε_E [(27) below] it reflects the undersaving of L and the exploitation of L by H studied in Sect. 2.

We generalize the standard deviations of the average consumption along the M paths. Let the expected average consumption in period t be denoted by c_t, therefore the squared standard deviations are respectively

$$\varepsilon_{t,E}^2 = \frac{1}{N}\sum_{k=1}^{N}(c_t^k - c_t)^2 \quad \text{and} \quad \varepsilon_{t,I}^2 = \frac{1}{NR}\sum_{k=1}^{N}\sum_{a=1}^{D}(c_{a,t}^k - c_t^k)^2. \tag{27}$$

We assume that every shortsighted worker k knows a small number of other shortsighted workers, indexed as $l \in L_k$.[3] We also assume that no set of acquaintances changes in time; the number of acquaintances is denoted by $|L_k| > 0, k = 1, 2, \ldots, N$. For simplicity, the acquaintances are just one period older than the foregoing worker. Except for Hs, every agent k at every age and time signals his current type $i(a, k, t+a)$.

Having dropped the age index, as a starting point, we shall experiment with the simplest network, described as follows. Let e be a positive integer, called *radius*, $0 < e \ll N$:

$$L_k^e = \{l, |l-k| \le e\},$$

where the N agents of the same cohort are allocated *randomly* on the circle with N points, and artificial types $N+1$, $N+2$, etc. stand for types 1, 2, etc. For example, the set of 1's acquaintances is

$$L_1^e = \{1, 2, 3, \ldots, e, e+1; N-e, N-e+1, \ldots, N-1, N\}.$$

We assume that worker (a, k) in period $t+a$ adopts that type $i_{a,k,t-1+a}$'s γ which produced the highest average *projected* consumption among his acquaintances one period earlier:

$$c_{a,k,t+a}^l \le c_{a,k,t+a}^{i_{a,k,t-1+a}}, \qquad l \in L_k. \tag{28}$$

If there is more than one optimal decision, he will pick one with the minimal index i or randomize.

To start the dynamic system at period 0, we have to define the initial conditions. For comparability with Tables 5, 6, 7, we assume that all the previous shortsighted savings were zero and those of the farsighted were $s^H(0)$ in ($4'$). Hoping that the process converges fast enough, we observe the system for $\mathbf{T} = 10$ time steps, which represents ten decades, i.e. 100 years.

[3] For the sake of simplicity, we forbid shortsighted workers to learn from farsighted workers but as suggested personally by Botond Kőszegi, its inclusion would open the door to an alternative learning mechanism not using government-made information like α and especially θ.

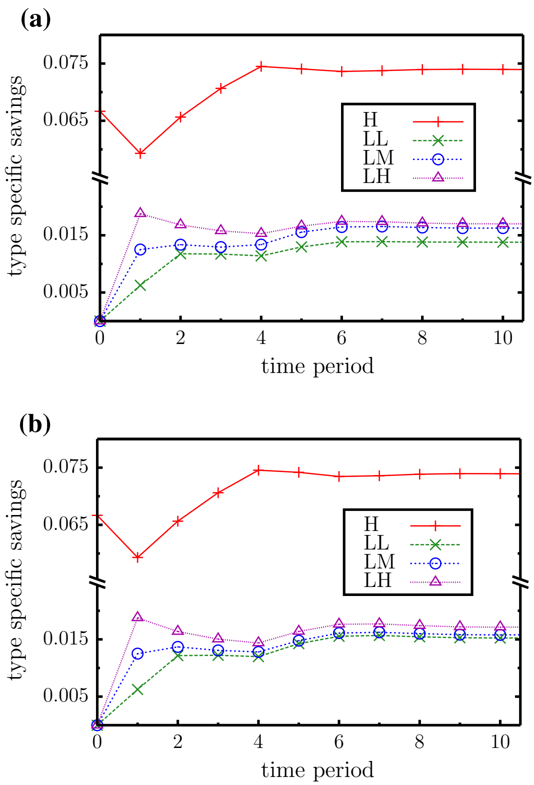

Fig. 1 **a** Rise of savings in time. **b** Rise of savings in time—averaged over 100 simulations

3.2 Numerical illustrations

We shall present eight illustrations.

3.2.1 Rise of savings in time

First the number of types is $n = 4$: with uniform frequencies $f_1 = f_2 = f_3 = f_4 = 1/4$ and with $\gamma_1 = 1/4$ (LL), $\gamma_2 = 1/2$ (LM) and $\gamma_3 = 3/4$ (LH), and type 4 is H. Note that the average relative saving propensity, $\gamma = 0.5$ is the same as in Table 5, therefore the two cases are comparable. Adding debt servicing, we choose $\zeta = 1/R$ in ($11'$). The number of workers at each cohort is $N = 120$ and $e = 1$, i.e. everybody has 3 acquaintances. The parameter values are $\mathbf{D} = 6$ and $\mathbf{R} = 4$, the available strategies are LL, LM, LH, and H, and L types do not learn from H types.

Figure 1a shows the average saving paths of workers who are initially of the same type. Note that on average, the middle shortsighted workers catch up with the higher ones, but the lower ones lag behind. The H types create "walls" that separate different types of behavior. Agents within such a domain can only become as smart as the

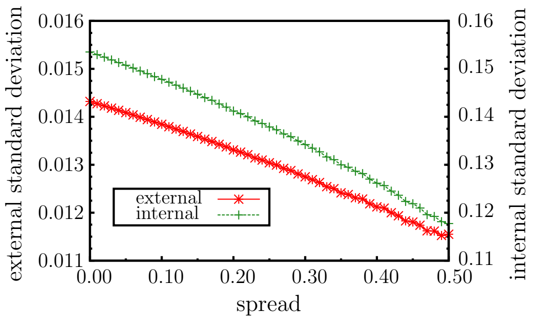

Fig. 2 Wider spread, typically smaller standard deviation of lifetime average consumption: $n = 4$

initially smartest agent was inside the domain. Despite learning, the average L-saving remains below 0.015, slightly less than previously.[4]

To check the robustness of our results, we repeated the simulation that lead to the results reported in Fig. 1a for 99 further randomly generated initial strategy distributions. At each time step, we averaged the savings, consumptions and their internal and external standard deviations over all 100 realisations. Figure 1b shows the average saving paths of workers who initially followed the same strategies after this statistical averaging. Corresponding standard deviations are all below 0.001, that is, they are at least one order of magnitude smaller than the plotted averages. As the two figures clearly show, the averaged results are very close to the data points produced by the previously mentioned single run. This—together with the relative smallness of deviations among different simulations—suggests that studying single simulation runs (with randomly generated initial states) should suffice to correctly capture the most important properties and describe the typical behaviour of our model system despite the above-mentioned domain effect. In the following illustrations, we present the results of single simulation runs.

3.2.2 The impact of rising spread in saving propensities

Next we continuously change the half distance between the extreme γs, that we call the *spread* ξ, while fixing the middle at $\gamma_2 = 1/2$: $\gamma_1 = 1/2 - \xi$ and $\gamma_3 = 1/2 + \xi$, $\xi \in (0, 1/2)$. Figure 2 (left-hand scale) shows the degree of perverse redistribution ε_E as a function of ξ, for a fixed $\mathbf{T} = 10$. We expected ε_E to decrease with ξ and Fig. 2 shows the extent of this tendency. Even in the case $\xi = 0.5$, the external standard deviation $\varepsilon_{E\mathbf{T}}$ cannot become zero, because of two reasons. Firstly, LL types do not save in their first working period. Secondly, the domain structure can prevent some LL type agents from becoming LH types. These reasons also lead to similar behavior in the internal standard deviation (right scale).

Comparing Table 4 (middle row) with the zero-spread outcome in Fig. 2, we arrive at the following observation: the introduction of local learning decreases the external

[4] Note that even extending the rules to allow learning from farsighted workers would not significantly improve learning. Upon request, we can send an unpublished document showing this.

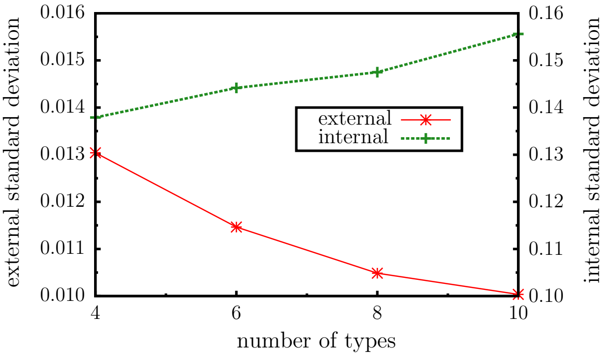

Fig. 3 More types, lower external standard deviation and higher internal standard deviation

standard deviation from 0.017 to 0.014, while it also increases the internal standard deviation from 0.121 to 0.155.

3.2.3 The impact of the number of types

In the third round, we increase the number of types from $n = 4$ to 6, 8 and 10. Retaining symmetry, we have $n = 2m$,

$$\gamma_i = \frac{i}{2m}, \qquad i = 1, 2, \ldots, m, \ldots, 2m - 1.$$

We again expect ε_E to decrease with m and Fig. 3 illustrates how our results match these expectations: as the learning process eventually turns most L types into LH types (with maximal γ), which become more and more similar to H types as m is increased. It is worth noting that the internal standard deviation (right scale) rises rather than falls.

3.2.4 Random acquaintances

Next we return to simpler systems, with fewer strategies. Again there are $n = 4$ strategies (H, LL, LM and LH), but this time we change the network's structure. We experiment with a random graph (Erdős and Rényi 1959): we replace the original connections with randomly chosen ones. Links were created with probability $p = \frac{2}{119}$, to ensure that the expected value of connections remains invariant. As Fig. 4 shows, the introduction of the random graph not only homogenizes the savings but also raises their average with respect to the case depicted in Fig. 1a.

3.2.5 Less acquaintances, less saving

What happens if we drastically lower the density of the better configured previous network by diminishing the probability of being connected from $2/119$ to $1/238$?

Fig. 4 The impact of different structure on the saving dynamics

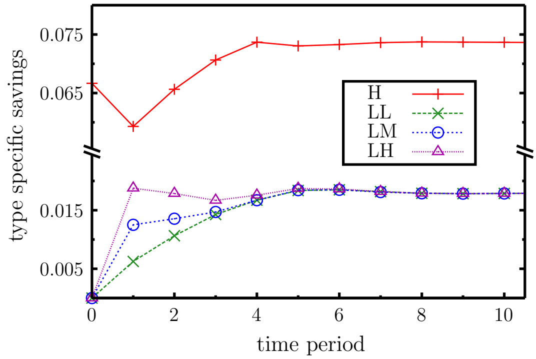

Fig. 5 The impact of less connections on the saving dynamics

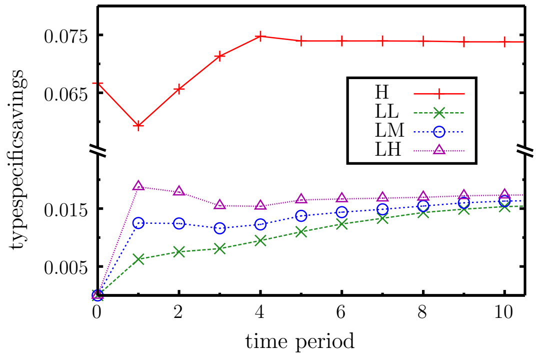

Figure 5 shows that this change leads to persistent heterogeneity and the stabilized average savings are also diminished.

3.2.6 More acquaintances, more saving

We also change the characteristics of the simpler, "cylindrical network", while fixing $m = 5$, $n = 10$. We increase the radius e from 1 to 2, 3 and 4, and expect $\varepsilon_{E,T}$ to increase with radius e. The parameter e controls the domain structure of the system, and therefore defines who can learn from whom. As e increases, more agents should become able to learn from L types with higher γ values, but this also changes the tax dynamics, so increasing e does not necessarily lead to a decrease in ε_E. Indeed, our calculations (omitted) show practically constant external standard deviations.

3.2.7 Frequent revision does not increase saving

Until now we have fixed the length of a time step period at $h = 10$ years. Now we change this important parameter as well. We expect that the shorter the length of periods, the faster the shortsighted agents will learn, diminishing both *relative* standard deviations. (We turn to relative standard deviations to neutralize the impact of changing time units.) Note, however, that our modifications change the debt dynamics as well,

Table 8 Standard deviations and length of period

Length of period (year) h	Internal standard deviation $\varepsilon_I(h)$	External standard deviation $\varepsilon_E(h)$
10	0.143	0.013
5	0.147	0.014
2	0.160	0.014
1	0.166	0.014

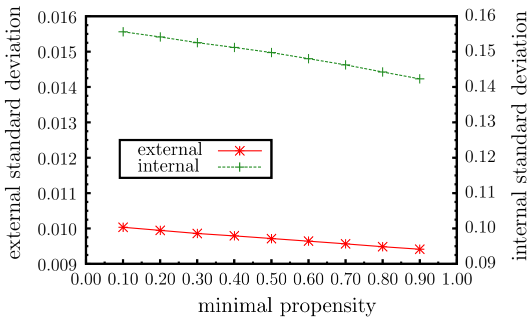

Fig. 6 Higher minimal propensity, lower standard deviation

which may have unexpected repercussions. To check our conjecture we diminish $h = 10$ first to 5 then to 2 and finally to 1. Table 8 displays the internal and external standard deviations at $T[1] = 100$ to be denoted as $\varepsilon_I(h)$ and $\varepsilon_E(h)$. Contrary to our conjecture, the speeding up of the learning process does not decrease the standard deviations, moreover, the internal standard deviation slightly increases.

3.2.8 Higher propensities to save: more saving

Finally, we investigated the effects of raising the average value of γ_i for a relatively high number of types. We fix the number of types at $n = 10$, and set the initial L-types' γ values to be equidistant above a minimal propensity $\gamma_{\min}$, ($\gamma_i = \gamma_{\min} + (i-1)\frac{1-\gamma_{\min}}{n-1}$, where $i = 1, 2, \ldots, n-1$). Figure 6 shows our findings. Raising the minimal propensity to save leads to a decrease in the standard deviations.

4 Conclusions

We have studied a family of utterly simple life-cycle/overlapping generations models with mandatory and voluntary pensions. In the preliminary first model, (passive) short- and farsighted workers lived together, and the former did not learn anything from their exploitation. In its full variant, the (active) shortsighted workers learned from public information how to participate in the voluntary pension system, and this diminished exploitation. In the second model, we added local learning to the use of public information: in the arising ABM, we studied a number of variants. For example, we observed a plausible outcome: the more heterogeneous the relative propensities to save, i.e. the richer the set of possibilities to learn, the lower the standard deviation of lifetime average consumptions, while increasing the number of acquaintances is indifferent. On the other hand, we observed an implausible outcome as well: regardless of the frequency of revision of individual strategies, the learning does not improve.

At the end we only allude to the shortcomings and simplifications present in our models. If we were to relax any of the above-mentioned constraints, the quantitative features would probably be changed. Though we have experimented with a number of variants and have a solid theoretical foundation, we can only hope that our most important qualitative result survives: local learning adds to using only public information. To have any empirical relevance, the next model should introduce wage heterogeneity, cap on matched savings and make farsightedness an increasing function of the wage.

Last but not least, the designers of voluntary pension systems should calculate with the effects of tax expenditures on matching. They should also take into account that the participation in such systems is difficult and the hindrances should be removed.

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